# An Exhaustive Covering Approach to Parameter-free Mining of Non-redundant Discriminative Itemsets 

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## Outline

- Background
- Our propsal
- Experiments


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- Background
- Our propsal - Experiments


## Background: Discriminative Patterns (1)

- Discriminative patterns:
- Show differences between two groups (classes)
- Used for:
- Characterizing the positive class
- Building more precise classifiers


Discriminative pattern $\boldsymbol{x}$


## Background: Discriminative Patterns (2)

- Discriminative patterns tend to be more meaningful than frequent patterns (thanks to class labels)
- Are class labels always available?
- Comparing groups is a standard starting point in data analysis
- Clustering can find groups (classes) $\rightarrow$ Cluster labeling

Clusters labeled with
Clusters discriminative patterns


## Background: Discriminative Patterns (3)

- Quality score: Measures the overlap between pattern $\boldsymbol{x}$ and positive class $c$


Quality is high


Quality is low

- Most of popular quality scores are not anti-monotonic: - Confidence, Lift
- Support difference, Weighted relative accuracy, Leverage
- F-score, Dice, Jaccard
$\rightarrow$ Branch \& bound pruning is often used [Morishita+ 00][Zimmarmann+ 09][Nijssen+ 09]


## Background: Coping with redundancy (1)

- Example: Item A is relevant to the positive class $\rightarrow$ Patterns containing A tend to be top-ranked in the candidate list (most of them are redundant)

|  | Dataset |  |  |
| :---: | :---: | :---: | :---: |
|  | TID | Class | Transaction |
| Positive Transactions | 1 | + | $\{A, B, D, E\}$ |
|  | 2 | $+$ | $\{A, B, C, D, E\}$ |
|  | 3 | + | $\{A, C, D, E\}$ |
|  | 4 | + | $\{A, B, C\}$ |
|  | 5 | + | \{B\} |
| Negative Transactions | 6 | - | $\{A, B, D, E\}$ |
|  | 7 | - | $\{B, C, D, E\}$ |
|  | 8 | - | $\{C, D, E\}$ |
|  | 9 | - | $\{\mathrm{A}, \mathrm{D}, \mathrm{E}\}$ |
|  | 10 | - | $\{\mathrm{A}, \mathrm{D}\}$ |

Top-15 patterns (+1 due to tie score)

| Rank | Pattern | F-score | TIDs Covered |
| :---: | :---: | :---: | :---: |
| 1 | $\{A, C\}$ | 0.75 | 2, 3, 4 |
| 2 | \{B\} | 0.73 | $1,2,4,5$ |
| 3 | \{A\} | 0.67 | $1,2,3,4$ |
| 3 | $\{A, B\}$ | 0.67 | 1,2,4 |
| 5 | $\{A, D, E\}$ | 0.60 | 1,2,3 |
| 5 | $\{\mathrm{A}, \mathrm{E}\}$ | 0.60 | $1,2,3$ |
| 5 | \{C\} | 0.60 | 2, 3, 4 |
| 8 | $\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ | 0.57 | 2, 4 |
| 8 | $\{\mathrm{A}, \mathrm{C}, \mathrm{D}\}$ | 0.57 | 2, 3 |
| 8 | $\{\mathrm{A}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$ | 0.57 | 2,3 |
| 8 | $\{A, C, E\}$ | 0.57 | 2, 3 |
| 12 | $\{\mathrm{A}, \mathrm{D}\}$ | 0.55 | $1,2,3$ |
| 13 | $\{A, B, D\}$ | 0.50 | 1,2 |
| 13 | $\{\mathrm{A}, \mathrm{B}, \mathrm{D}, \mathrm{E}\}$ | 0.50 | 1,2 |
| 13 | $\{A, B, E\}$ | 0.50 | 1,2 |
| 13 | $\{\mathrm{B}, \mathrm{C}\}$ | 0.50 | 2, 4 |

- Set-inclusion-based constraints
- Closedness [Pasquier+ 99]
- Productivity [Bayardo 00][Webb 07]


## Background: Coping with redundancy (2) <br> - Set-inclusion-based constraints

- Closedness [Pasquier+ 99]
- Productivity [Bayardo 00][Webb 07]

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| 1 | $\{A, C\}$ | 0.75 | 2,3,4 |  |  |
| 2 | \{B\} | 0.73 | 1, 2, 4, 5 |  | Closedness: |
| 3 | \{A\} | 0.67 | $\frac{1}{1}, 2,3,4$ |  | For patterns covering |
| $\begin{aligned} & 3 \\ & 5 \end{aligned}$ | $\{A, B\rangle$ $\{A, D, E\}$ | $0.67$ | $1,2,4$ |  | the same (positive) |
| 5 | $\{A, E\}$ | 0.60 | 1, 2, 3 |  | transactions, |
| 5 | \{C\} | 0.60 | 2, 3, 4 |  | pick the largest one |
| 8 | $\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ | 0.57 | 2, 4 |  |  |
| 8 | $\{A, C, D\}$ | 0.57 | 2, 3 |  |  |
| 8 | $\{\mathrm{A}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$ | 0.57 0.57 | 2, 3 |  |  |
| 8 | $\{A, C, E\}$ $\{A, D\}$ | 0.57 | 2, ${ }^{2,2} 2,3$ |  |  |
| 13 | $\{A, B, D\}$ | 0.50 | 1,2 |  |  |
| 13 | $\{\mathrm{A}, \mathrm{B}, \mathrm{D}, \mathrm{E}\}$ | 0.50 | 1,2 |  |  |
| 13 | $\{\mathrm{A}, \mathrm{B}, \mathrm{E}\}$ | 0.50 | 1,2 |  |  |
| 13 | $\{\mathrm{B}, \mathrm{C}\}$ | 0.50 | 2, 4 |  |  |

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| 3 | [A] | 0.67 0.67 | 1, 2, 3, 4 | For patterns covering |
| 3 5 | \{A, $\left\{\begin{array}{l}\text { a } \\ \text {, }\end{array}\right.$ | 0.67 0.60 | $1,2,4$ $1,2,3$ | the same (positive) |
| 5 | $\{A, E\}$ | 0.60 | 1, 2, 3 | transactions, |
| 8 | \{A, B, C\} | 0.65 | le $\begin{aligned} & 1,3,4 \\ & 2,4\end{aligned}$ | lar |
| 8 | \{A, C, D $\}$ | 0.57 | 2, 3 |  |
| 8 | $\{A, C, D, E\}$ | 0.57 | 2, 3 |  |
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| 5 | $\{A, E\}$ | 0.60 | 1,2,3 |
| 5 | \{C\} | 0.60 | 2, 3, 4 |
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| 8 | $\{A, C, D\}$ | 0.57 | 2,3 |
| 8 | $\{A, C, D, E\}$ | 0.57 | 2, 3 |
| 8 | $\{A, C, E\}$ | 0.57 | 2,3 |
| 12 | $\{A, D$, | 0.55 | $1,2,3$ |
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| 13 | $\{A, B, D, E\}$ | 0.50 | 1,2 |
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16 patterns $\rightarrow \mathbf{8}$ patterns

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## Productivity:

If a super-pattern has no higher quality, remove it

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| 3 | \{A\} | 0.67 | 1, 2, 3, 4 |
| 3 | A, B $\}$ | 0.67 | 1,2,4 |
| 5 | $\{\mathrm{A}, \mathrm{D}, \mathrm{E}\}$ | 0.60 | 1, 2, 3 |
| 5 | $\{\mathrm{A}, \mathrm{E}\}$ | 0.60 | 1, 2, 3 |
| 5 | \{C\} | 0.60 | 2, 3, 4 |
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| 8 | $\{A, C, D\}$ | 0.57 | 2,3 |
| 8 | $\{\mathrm{A}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$ | 0.57 | 2, 3 |
| 8 | $\{\mathrm{A}, \mathrm{C}, \mathrm{E}\}$ | 0.57 | 2, 3 |
| 12 | \{A, D\} | 0.55 | 1,2,3 |
| 13 | $\{\mathrm{A}, \mathrm{B}, \mathrm{D}\}$ | 0.50 | 1,2 |
| 13 | \{A, B, D, E\} | 0.50 | 1,2 |
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| 3 | $\{A$, | 0.67 | $1,2,3,4$ |
| 3 | $\{A, B\}$ | 0.67 | $1,2,4$ |
| 5 | $\{A, D, E\}$ | 0.60 | $1,2,3$ |
| 5 | $\{A, E\}$ | 0.60 | $1,2,3$ |
| 5 | $\{C\}$ | 0.60 | $2,3,4$ |
| 8 | $\{A, B, C\}$ | 0.57 | 2,4 |
| 8 | $\{A, C, D\}$ | 0.57 | 2,3 |
| 8 | $\{A, C, D, E\}$ | 0.57 | 2,3 |
| 8 | $\{A, C, E\}$ | 0.57 | 2,3 |
| 12 | $\{A, D\}$ | 0.55 | $1,2,3$ |
| 13 | $\{A, B, D\}$ | 0.50 | 1,2 |
| 13 | $\{A, B, D, E\}$ | 0.50 | 1,2 |
| 13 | $\{A, B, E\}$ | 0.50 | 1,2 |
| 13 | $\{B, C\}$ | 0.50 | 2,4 |

16 patterns $\rightarrow \mathbf{4}$ patterns

## Background: Coping with redundancy (2)

- Set-inclusion-based constraints
- Productivity + Closedness [Kameya+ 13]

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| 3 | $\{A, B\}$ | 0.67 | $1,2,4$ |
| 5 | $\{A, E, E\}$ | 0.60 | $1,2,3$ |
| 5 | $\{A, E\}$ | 0.60 | $1,2,3$ |
| 55 | $\{C\}$ | 0.60 | $2,3,4$ |
| 8 | $\{A, B, C\}$ | 0.57 | 2,4 |
| 8 | $\{A, C, D\}$ | 0.57 | 2,3 |
| 8 | $\{A, C, D, E\}$ | 0.57 | 2,3 |
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| 12 | $\{A, D\}$ | 0.55 | $1,2,3$ |
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16 patterns $\rightarrow \mathbf{3}$ patterns

## Background: Coping with redundancy (3)

- The best-covering constraint
- In the same spirit of the HCC (highest confidence covering) constraint in HARMONY [Wang+ 05]

| Rank | Pattern | F-score | TIDs Covered | Best-covering: |
| :---: | :---: | :---: | :---: | :---: |
| 1 | \{A, C $\}$ | 0.75 | 2, 3, 4 | ) Every pattern must be |
| 2 | \{B\} | 0.73 | 1, 2, 4, 5 | \} the best to at least one |
| 3 | \{A\} | 0.67 | 1, 2, 3, 4 | positive transaction |
| 3 | $\{\mathrm{A}, \mathrm{B}\}$ | 0.67 | 1, 2, 4 |  |
| 5 | \{A, D, E\} | 0.60 | 1, 2, 3 |  |
| 5 | $\{\mathrm{A}, \mathrm{E}\}$ | 0.60 | 1,2,3 |  |
| 5 | \{C\} | 0.60 | 2, 3, 4 |  |
| 8 | $\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ | 0.57 | 2, 4 |  |
| 8 | \{A, C, D\} | 0.57 | 2, 3 |  |
| 8 | $\{\mathrm{A}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$ | 0.57 | 2, 3 |  |
| 8 | $\{\mathrm{A}, \mathrm{C}, \mathrm{E}\}$ | 0.57 | 2,3 |  |
| 12 | \{A, D $\}$ | 0.55 | 1,2,3 |  |
| 13 | \{A, B, D $\}$ | 0.50 | 1,2 |  |
| 13 | $\{A, B, D, E\}$ | 0.50 | 1,2 |  |
| 13 | $\{A, B, E\}$ | 0.50 | 1,2 |  |
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| 2 | S $\left.{ }^{\text {S }}\right\}$ | 0.73 | $1,2,4,5$ | \} the best to at least one |
| 3 | \{A\} | 0.67 | $1,2,3,4$ | positive transaction |
| 3 | \{A, B $\}$ | 0.67 | 1, 2, 4 | positive transaction |
| 5 | $\{\mathrm{A}, \mathrm{D}, \mathrm{E}\}$ | 0.60 | $1,2,3$ |  |
| 5 | $\{A, E\}$ | 0.60 | 1,2,3 |  |
| 5 | \{C\} | 0.60 | 2, 3, 4 |  |
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| 8 | $\{A, C, D\}$ | 0.57 | 2, 3 |  |
| 8 | \{A, C, D, E\} | 0.57 | 2,3 |  |
| 8 | \{A, C, E $\}$ | 0.57 | 2, 3 |  |
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| 3 | \{A, B $\}$ | 0.67 | 1, 2, 4 |
| 5 | \{A, D, E\} | 0.60 | 1,2,3 |
| 5 | $\{\mathrm{A}, \mathrm{E}\}$ | 0.60 | 1,2,3 |
| 5 | \{C\} | 0.60 | 2, 3, 4 |
| 8 | \{A, B, C\} | 0.57 | 2, 4 |
| 8 | $\{A, C, D\}$ | 0.57 | 2, 3 |
| 8 | $A \mathrm{~A}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$ | 0.57 | 2,3 |
| 8 | $\{A, C, E\}$ | 0.57 | 2,3 |
| 12 | \{A, D\} | 0.55 | $1,2,3$ |
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16 patterns $\rightarrow \mathbf{2}$ patterns

## Background: Control parameters

- Minimum support (minsup) $\sigma_{\min }$ is a sensitive control parameter
- Top-k mining [Han+ 02]:

$-k=$ "\# of output patterns"
$-k$ is fairly easy to specify because we usually know how many patterns we can handle ( $k$ is more human-centric than $\sigma_{\text {min }}$ )
- However, we do not exactly know in advance how many useful patterns we can mine
- Is it possible to remove even $k$ ?


## Background: Sequential covering (1)

- Sequential covering:
- One traditional way for building a rule-based classifier
- Procedure:
- Iterate until there are no uncovered positive examples
- Induce a new rule $r$
- Remove all positive examples covered by $r$


## Positive examples <br> Negative examples

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Positive examples

Negative examples


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Positive examples

Negative examples


## Background: Sequential covering (2)

- Problems in removing positive examples:
- Lately-generated rules may not be meaningful
- The number of positive examples decreases [Domingos 94] $\rightarrow$ Lately-generated rules may not be statistically reliable

Next rules must be learned
Positive examples

Negative examples

## Our proposal

- ExCover: an efficient and exact method for finding non-redundant discriminative itemsets
- Features:
- Exhaustive search unlike sequential covering
- Best-covering constraint tighter than productivity $\rightarrow$ fewer redundant patterns
- No control parameters limiting the search space


## Outline

$\checkmark$ Background

- Our proposal
- Best-covering constraint - ExCover
- Experiments


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## Best-covering constraint (1)

- Best-covering constraint: "Every pattern must have the highest quality for at least one positive transaction it covers"



## Best-covering constraint (2)

- Best-covering constraint: "Every pattern must have the highest quality for at least one positive transaction it covers"



## Best-covering constraint (3)

- Best-covering constraint: "Every pattern must have the highest quality for at least one positive transaction it covers"



## Best-covering constraint (3)

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## Best-covering constraint (3)

- Best-covering constraint:
"Every pattern must have the highest quality for at least one positive transaction it covers"


Positive transactions

We can also say : $x$ must have higher quality than any other competitors for some instance

Instance of $x$

## Best-covering constraint (4)

- Tightness:

Best-covering is tighter than productivity
Sketch of proof

- Sub-pattern of $\boldsymbol{x}$ is always a competitor of $\boldsymbol{x}$
- If $\boldsymbol{x}$ is best-covering, its sub-pattern must have lower quality
- Productivity: $\boldsymbol{x}$ must have higher quality than its sub-patterns


## Best-covering:

$x$ must have higher quality than any other competitors for some instance

- Branch \& bound pruning:

We can safely prune $\boldsymbol{x}$ and its descendants when the upper bound of $x$ 's quality is lower than the quality of any competitor of $x$

## Outline

## $\checkmark$ Background

- Our proposal


# $\checkmark$ Best-covering constraint 

- ExCover
- Experiments


## ExCover: Search space

- Basic strategy:
- Depth-first search by a variant [kameya+ 13] of LCM [Uno+ 04]:
- Only visits patterns closed on positive transactions $\rightarrow$ The closedness constraint is built-in
- Visits earlier shorter patterns including high quality items $\rightarrow$ There is more chance of pruning

Enumeration tree
of closed patterns:

All combinations
of $B$ and $A$$\{B, A\}\{B, A, C\}\{B, A, E, D\}\{A, C, E, D\}$
All combinations
of $B, A$ and $C$
$\{B, A, C, E, D\}$
Quality of item:
B $>\mathrm{A}>\mathrm{C}>\mathrm{E}>\mathrm{D}$

## ExCover: Candidate table

- Basic strategy (cont'd):
- Top-1 (Top- $k$ with $k=1$ ) mining concurrently for each positive transaction
- Candidate patterns are maintained in the candidate table following the best-covering constraint


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## ExCover: Candidate table

- Basic strategy (cont'd):
- Top-1 (Top- $k$ with $k=1$ ) mining concurrently for each positive transaction
- Candidate table is a map:

Positive transaction $t \rightarrow$ Best competitor(s) for $t$
Candidate table

Current pattern


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Candidate table

Current pattern

| $t_{1}$ |  |
| :--- | :--- |
| $t_{2}$ |  |
| $t_{3}$ | (empty) |
| $t_{4}$ |  |
| $t_{5}$ |  |
| $t_{6}$ |  |
| $t_{7}$ | $z_{1}$ |
| $t_{8}$ |  |
| $t_{9}$ | $z_{2}$ |

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- Top-1 (Top- $k$ with $k=1$ ) mining concurrently for each positive transaction
- Candidate table is a map:

Positive transaction $t \rightarrow$ Best competitor(s) for $t$ Candidate table

Current pattern


Quality $(\boldsymbol{x})>\operatorname{Quality}\left(\boldsymbol{z}_{1}\right)$

## ExCover: Candidate table

- Basic strategy (cont'd):
- Top-1 (Top- $k$ with $k=1$ ) mining concurrently for each positive transaction
- Candidate table is a map:

Positive transaction $t \rightarrow$ Best competitor(s) for $t$ Candidate table

Current pattern


## ExCover: Candidate table

- Basic strategy (cont'd):
- Top-1 (Top- $k$ with $k=1$ ) mining concurrently for each positive transaction
- Candidate table is a map:

Positive transaction $t \rightarrow$ Best competitor(s) for $t$ Candidate table

Current pattern

$\operatorname{Quality}(\boldsymbol{x})<\operatorname{Quality}\left(\boldsymbol{z}_{2}\right)$

## ExCover: Candidate table

- Basic strategy (cont'd):
- Top-1 (Top- $k$ with $k=1$ ) mining concurrently for each positive transaction
- Candidate table is a map:

Positive transaction $t \rightarrow$ Best competitor(s) for $t$ Candidate table

Current pattern


## ExCover: Candidate table

- Basic strategy (cont'd):
- Top-1 (Top- $k$ with $k=1$ ) mining concurrently for each positive transaction
- Candidate table is a map:

Positive transaction $t \rightarrow$ Best competitor(s) for $t$ Candidate table

Pruned!

Current pattern
$\overline{\mathrm{Quality}(\boldsymbol{x})}<\mathrm{Quality}\left(\boldsymbol{z}_{1}\right)$
upper bound of $x$ 's quality
$\overline{\text { Quality }(\boldsymbol{x})}<$ Quality $\left(z_{2}\right)$
$\overline{\operatorname{Quality}(\boldsymbol{x})}<\operatorname{Quality}\left(z_{3}\right)$

## ExCover: Property

- ExCover is...
- Exhaustive
- Only performs safe branch \& bound pruning
- Parameter-free
- Conducts concurrent top-1 mining

Fixed inside the algorithm

## ExCover: Related work

- HARMONY [Wang+ 05]
- Uses the same strategy as that of ExCover
- However its original paper does not mention on redundancy
- Uses confidence $p(c \mid \boldsymbol{x})$ as the quality score
- Confidence prefers highly specific patterns $\rightarrow$ Not easy to have its upper bound
- User-specified minsup $\sigma_{\min }$ is required for pruning


## Outline

$\checkmark$ Background
$\checkmark$ Our proposal
$\checkmark$ Best-covering constraint $\checkmark$ ExCover

- Experiments


## Experiments: Outline

- We use datasets from UCI ML Repository
- Experiment 1:
- Detailed analysis on redundancy among patterns using the Mushroom dataset
- Experiment 2:
- Analysis on search performance using 16 datasets preprocessed by the CP4IM project:

| Dataset | \#Trans. | \# Items |
| :--- | ---: | ---: |
| anneal | 812 | 93 |
| audiology | 216 | 148 |
| australian-credit | 653 | 125 |
| german-credit | 1,000 | 112 |
| heart-cleveland | 296 | 95 |
| hepatitis | 137 | 68 |
| hypothyroid | 3,247 | 88 |
| kr-vs-kp | 3,196 | 73 |


| Dataset | \#Trans. | Items |
| :--- | ---: | ---: |
| lymph | 148 | 68 |
| mushroom | 8,124 | 110 |
| primary-tumor | 336 | 31 |
| soybean | 630 | 50 |
| splice-1 | 3,190 | 287 |
| tic-tac-toe | 958 | 28 |
| vote | 435 | 48 |
| zoo-1 | 101 | 36 |

## Experiment 1: Mushroom

## Covers

4,112 out of
4,208 positive transactions
Productivity + Closedness + Top- $k$ [Kameya+ 13] ( $k=30$ )

| Rank | Pattern | F-score |
| :---: | :--- | :---: |
| 1 | \{odor=n, veil-type=p\} | 0.881 |
| 2 | \{gill-size=b, stalk-surface-above-ring=s, veil-type=p\} | 0.866 |
| 3 | \{gill-size=b, stalk-surface-below-ring=s, veil-type=p\} | 0.837 |
| 4 | \{gill-size=b, veil-type=p\} | 0.798 |
| 5 | \{stalk-surface-above-ring=s, veil-type=p\} | 0.776 |
| 6 | \{ring-type=p, veil-type=p\} | 0.771 |
| 7 | \{stalk-surface-below-ring=s, veil-type=p\} | 0.744 |
| 8 | $\{$ veil-type $=$ p $\}$ | 0.682 |

Covers remaining
96 positive transactions

## Experiment 1: Mushroom

 loses information from 96 positive transactions!Productivity + Closedness + Top- $k$ [Kameya+ 13] ( $k=30$ )

|  | Rank | Pattern | F-score |
| :---: | :---: | :---: | :---: |
|  | 1 | \{odor=n, veil-type=p\} | 0.881 |
| $\longrightarrow$ | 2 | \{gill-size=b, stalk-surface-above-ring=s, veil-type=p\} | 0.866 |
|  | 3 | \{gill-size=b, stalk-surface-below-ring=s, veil-type=p\} | 0.837 |
|  | 4 | \{gill-size=b, veil-type=p\} | 0.798 |
|  | 5 | \{stalk-surface-above-ring=s, veil-type=p\} | 0.776 |
|  | 6 | \{ring-type=p, veil-type=p\} | 0.771 |
|  | 7 | \{stalk-surface-below-ring=s, veil-type=p\} | 0.744 |
|  | 8 | \{veil-type=p\} | 0.682 |
|  Covers rema <br> ExCover 96 positive |  |  |  |
|  | Rank | Pattern | F-score |
|  | 1 | \{odor=n, veil-type=p\} | 0.881 |
|  | 2 | \{gill-size=b, stalk-surface-above-ring=s, veil-type $=$ p\} | 0.866 |
|  | 3 | \{stalk-surface-above-ring=s, veil-type=p\} | 0.776 |

We only need 3 best-covering patterns

## Experiment 2: Settings

- 16 datasets preprocessed by the CP4IM project
- Previous method in comparison [Kameya+ 13]:
- Productivity + Closedness + Top-k
$-k$ was chosen from 10, 100 and 1,000


## Experiment 2: \#Patterns

- ExCover outputs a more compact set of patterns
- \# of output patterns was moderate and did not vary


## Experiment 2: Search space

- Search space = \# of visited patterns in depth-first search



## Experiment 2: Running time

- Our implementation: In Java
- Running time averaged over 30 runs
- For most datasets, ExCover finishes within one second


## Summary

- ExCover: an efficient and exact method for finding non-redundant discriminative itemsets
- Works under the best-covering constraint
- Requires no control parameters limiting the search space
- Finds a more compact set of patterns in a shorter time


## Future work

- Transactions including numeric values
- Building classifiers from best-covering patterns
- Sequence pattern mining

